NUMERICAL STUDIES OF THE CURVED BEAM PROBLEM

BY

HSUN-KUANG YUAN

B.S., St. John's University, 1945
M.S., University of Illinois, 1948

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
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NUMERICAL STUDIES OF THE CURVED BEAM PROBLEM

While beams with slight curvatures may be analyzed, with very little error, by means of the familiar straight beam formulas, curved beams — defined as flexural members with relatively sharp curvatures — present problems which cannot be solved by the said formulas without introducing considerable error. Therefore curved beams require separate treatments.

Exact solutions by the theory of elasticity have been given for a few simple problems, but many other problems that are more complicated remain as yet unsolved. The two well-known approximate theories, the Winkler-Bach theory and the Andrews-Pearson theory, can be used to solve the majority of curved beam problems with satisfactory results, but, theoretically speaking, they are both limited in application because they deal only with symmetrical cases. It is mainly to overcome this difficulty that the present investigation was undertaken.

A theory of flexure of curved beams is developed by which flexural stresses in curved beams due to bending about two axes, one radial and the other perpendicular to the plane of curvature, can be readily determined. It is based upon the assumptions of plane transverse sections remaining plane and longitudinal fibers being in simple tension or compression.

In the present method of analysis of curved beams, designated as "the method of transformed sections", the cross section of the beam is first transformed by multiplying each differential area in the section by its "density factor", which is the ratio of the radius of curvature of the innermost fibers to that of the fiber in question. In other words, each differential area in the original section is made "porous", with an effective or solid area equal to the density factor times the original differential area. Thus the transformed section may

be pictured as one having no holes or pores at the innermost fibers but becoming more and more porous toward the outermost fibers. This procedure follows a suggestion published by Professor Hardy Cross.

From the conditions of equilibrium, the fictitious stress at any point in the transformed section is then expressed in terms of the external forces and the properties of the transformed section. Finally, the actual stress at the corresponding point in the original section is obtained by multiplying the fictitious stress by the density factor of the fiber.

Unlike the Winkler-Bach and other formulas, the formulas derived here do not involve the difference of two very nearly equal quantities and, therefore, when working with the method of transformed sections, one does not have to carry calculations to a large number of significant figures to obtain results that are sufficiently accurate for all practical purposes. Furthermore, these formulas are so similar to the corresponding ones for straight beams and the concept of the transformed section is so easy to grasp that the present method readily lends itself to quick understanding and mastery. Hence, one can solve curved beam problems as easily as straight beam problems by means of the method of transformed sections.

Two typical problems are solved to illustrate the method: one for an unsymmetrical case and the other for a symmetrical case. In the latter problem it is shown that the amount of numerical computations may be reduced considerably by a proper choice of divisions.

Stresses in curved beams due to shearing forces and twisting moments are not considered in this analysis.

Inasmuch as twisting of curved beams with thin-walled open cross sections is undesirable, a formula to locate the "shear center plane", the plane in which all external loads must act in order that torsion or twist is eliminated at all cross sections of the beam, is derived. In this derivation, use is made of some important relationsihps among the normal force,

the shearing forces, the bending moments, and the twisting moment, that act on any cross section of a curved beam, and the external loads on the beam. These relationships are also established in this investigation, together with a general formula for shear flow, due to bending only, in a thin-walled open cross section of a curved beam.

The said shear-center-plane formula is derived for certain types of channel sections. Other cross sections are briefly discussed. A third problem illustrates the straightforward application of the formula for channel sections.

The problem of secondary bending of flanges of thin-walled symmetrical I- and T-sections has been solved by Dr. Hans Bleich; however, for the sake of completeness of this investigation, it is presented briefly in the last part of this dissertation along with Bleich's solution in a summarized form. This solution consists in replacing the original cross section of a curved beam by an ideal cross section, having so-called effective flange widths that are narrower than the original ones. The effective widths may be computed from Poisson's ratio of the material and the dimensions of the original cross section. If the method of transformed sections is now applied to this ideal cross section to solve for the required stresses, the secondary bending effect will be taken care of automatically. A fourth problem is solved for an I-sectioned curved beam under pure bending in the plane of the web by end couples. This example indicates that a considerable increase in the maximum longitudinal flexural stress may result from the secondary bending of the flanges.

VITA

Hsun-kuang Yuan was born on June 10, 1926, in Ningpo, Chekiang, China. After receiving his high school education at St. John's Middle School, Shanghai, China, he entered St. John's University, Shanghai, China, in February, 1943, and in September, 1945, he graduated there with the degree of Bachelor of Science, in Civil Engineering. In June, 1946, he was awarded the "Best Civil Engineering Graduate of 1945" medal for his achievement of the highest undergraduate scholastic average among the civil engineering graduates of the academic year 1945-46. From October, 1945, to August, 1947, he served as an instructor in the Department of Civil Engineering of his Alma Mater.

He began his graduate study at the University of Illinois in September, 1947, and received the degree of Master of Science in Civil Engineering in June, 1948. From February, 1948, to June, 1950, he held a Graduate Fellowship in the Department of Civil Engineering, and in September, 1950, he was appointed Research Graduate Assistant in the same department, a position which he now holds.

In the summer of 1948, he worked on the design of bridges at Parsons, Brinckerhoff, Hall and Macdonald, Engineers, New York City; and in the summers of 1949 and 1950, he was engaged in the design of buildings at Strobel and Salzman, Consulting Engineers, New York City, and at Luria Engineering Corporation, Bethlehem, Pennsylvania, respectively.

He was elected an associate member of Sigma Xi in December, 1948, a member of Phi Kappa Epsilon in May, 1949, and a member of Pi Mu Epsilon in May, 1950.

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